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HEAT TRANSFER INSIDE A ROTATING RECTANGULAR CAVITY

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The qualitative liquid-flow pattern is considered for a rotating rectangular cavity in the case of heat transfer in a direction parallel to the axis of rotation; calculational relations describing the heat transfer are obtained.

Calculations of the temperature state of rotating machine parts require information on heat-transfer coefficients in channels and cavities. One type of rotating cavity is shown in Fig. 1: The cavity is in the form of a rectangular closed parallelepiped, through which there is heat transfer in a direction parallel to the axis of rotation. The liquid motion in such a cavity is determined by centrifugal and Coriolis forces.

The field due to the centrifugal mass forces is inhomogeneous, because of the change in the liquid density in the thermal boundary layer near the heat-transfer surface and also as a result of the variation in the inertial centrifugal acceleration over the radius of rotation.

The variation in liquid density in the direction normal to the heat-transfer surface gives rise to radial displacement of the liquid and leads to the development of circulation in opposite directions in the two halves of the cavity. The directions of the circulatory motion are indicated on the side surface of the cavity in Fig. 1.

The liquid motion near the cooled wall occurs in a positive pressure gradient, while close to the heated wall there is a negative pressure gradient.

The Coriolis acceleration vector lies in the cross-sectional plane, and varies in magnitude in accordance with the velocity of radial displacement of the liquid. The distribution of the radial velocity and the direction of the Coriolis force in the cross section are indicated on the front wall of the cavity in Fig. 1. This Coriolis-force distribution may give rise to rotational motion of the liquid, leading to changes in the heat-transfer conditions. The direction of motion is shown in Fig. 1.

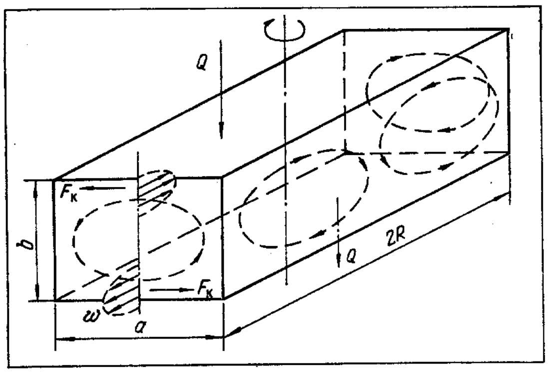

Fig. 1. Diagram of rotating cavity.

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The other feature of the rotating system is associated with the effect of the mass-force field on the stability of liquid motion. When the vectors F and grad |F| coincide, the mass forces stabilize the motion; they hinder the transition from laminar to turbulent flow or reduce the intensity of turbulent motion. In the opposite case, the mass forces facilitate the development of perturbations and the appearance of secondary flows [1].

In this system, the variation in the mass forces is complex. The primary causes of radial displacement of the liquid are the liquid-density gradient, the vector of which is perpendicular to the heat-transfer surface, and the centrifugal mass force directed along the radius of rotation. Therefore for each cross section the vectors F_c and grad ρ are mutually perpendicular; this creates conditions in which the inhomogeneous field of centrifugal mass forces is able to produce liquid motion. However, the centrifugal acceleration varies from cross section to cross section, and for ρ = const the vectors F_c and grad $|F_c|$ come to coincide, resulting in a stabilizing effect of the mass forces on the flow. This effect is another difference between rotating and fixed systems.

Theoretical investigation of the problem under consideration is difficult, both because of the complexity of the mass-force field determining the liquid motion and, in particular, because it is impossible to take into account the changes in heat transfer due to the stabilizing effect. There has been an experimental investigation of heat transfer in a cavity with 2R = 279.5 mm, a = 25.4 mm, and b = 26.2 mm (notation as in Fig. 1) [2]. The cavity was filled with silicone oils with Pr = 7 and 3000, and rotated around a vertical axis at frequencies of 264-565 rpm.

Also in [2], an attempt was made to generalize these experiments on the basis of the Rossby and Ekman numbers, which do not include the inertial acceleration determining the intensity of free motion in the system. The determining dimension was taken to be the distance between the cooled and heated surfaces, although the maximum thickness of the laminar boundary layer, calculated using the inertial centrifugal acceleration in various conditions, was found to be 2.4-24% of this distance; hence the boundary layers do not interact. It proved impossible to generalize the results of experiments for the two values of Pr.

In the present work, the experimental data of [2] are analyzed in the form of a similarity equation

$$\mathrm{Nu}^*_m = c\mathrm{Ra}^m_m \mathrm{Pr}^n_m, \tag{1}$$

in which the Nusselt number is written in terms of the total heat-transfer coefficient

$$\alpha^* = \frac{1}{\dfrac{1}{\alpha_1} + \dfrac{1}{\alpha_2}}, \tag{2}$$

and the Rayleigh number is based on the centrifugal acceleration calculated at the outer radius:

$$j_c = \omega^2 R. \tag{3}$$

The ratio of absolute liquid temperatures in [2] differed from unity by no more than 3%. Accordingly, Eq. (1) does not include a parameter reflecting the effect of nonisothermal conditions on the physical properties of the liquid.

The number Ra = GrPr is a generally accepted similarity parameter reflecting the intensity of free convection; however, it does not take the effect of the physical properties of the liquid completely into account and so the Prandtl number is also included in Eq. (1). As shown by the theoretical and experimental investigation of [3], $n > 0$.

Note that Eq. (1) does not include a parameter taking into account the effect of the Coriolis forces on the heat transfer. This is because the radial velocity of the liquid, which determines the Coriolis acceleration, does not appear in the boundary conditions of the problem. In addition, this velocity depends on the centrifugal acceleration and the variation in liquid density in the system, which is taken into account by Ra. Therefore, in the problem under consideration, the value of Ra indirectly reflects the effect of Coriolis forces on the heat transfer.

In Eq. (1), the determining temperature was taken to be the mean liquid temperature in the cavity, $t_m = (t_1 + t_2)/2$ and the determining dimension is taken to be the maximum radius of rotation R. This choice satisfactorily characterizes the heat-transfer conditions for free convection in the cavity when $\delta_{cw} \ll b$.

The results of generalizing experimental data are shown in Fig. 2. The line in Fig. 2 corresponds to the equation

$$\mathrm{Nu}^*_m = 0.058\,\mathrm{Ra}_m^{0.3}\,\mathrm{Pr}_m^{0.078}. \tag{4}$$

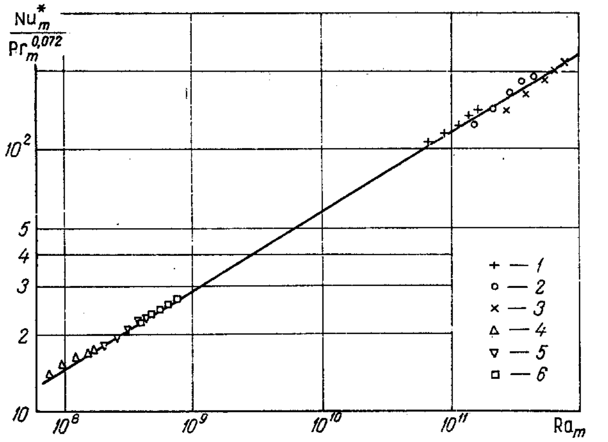

Fig. 2

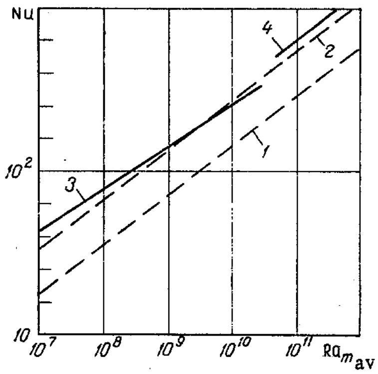

Fig. 3

Fig. 2. Results of generalizing experimental data: 1-3) Pr = 7; ω = 27.7, 45, and 59.2 rad/sec; 4-6) Pr = 3000; ω = 27.7, 45, and 59.2 rad/sec.

Fig. 3. Comparison of heat-transfer intensities under free convection in inertial and gravitational mass-force fields: 1, 2) from Eq. (5) for Pr = 1 and 3000; 3, 4) data of [4] for a fixed vertical wall with a laminar and a turbulent boundary layer.

The greatest deviation of the points from this line is 12% but for 90% of the points the deviation is no more than 5%. Equation (4) generalizes the experimental data for Ra = 10^8–10^{12} and Pr = 7–3000.

For a fixed vertical surface, according to the data of [4], turbulent conditions begin to develop at Ra ≅ $6 \cdot 10^{10}$ and the corresponding power to which Ra must be raised is m = 0.33; at Ra < $6 \cdot 10^{10}$ there is laminar flow characterized by m = 0.25.

It is evident from Fig. 2 that for a rotating cavity m = 0.3 and remains constant over the whole range of Ra investigated. This is evidently because the stabilizing effect of the mass-force field prevents the appearance of turbulence in the experimental conditions investigated and the liquid remains laminar. Similar results have been found for the heat transfer in a closed cavity between rotating disks [5]. The increase in m in comparison with the case of motion in fields of gravitational mass forces may be attributed to the distinctive features of the rotating system. In particular, it may be associated with the effect of Coriolis acceleration on the liquid motion and heat transfer.

The results obtained for heat transfer in a rotating cavity will now be compared with the heat transfer at a vertical wall in a gravitational field. For this purpose, Ra in Eq. (4) must be expressed in terms of the inertial acceleration at the mean radius and Nu* must be expressed in terms of the heat-transfer coefficient at the heat-transfer surface. If it is assumed that the mean heat-transfer coefficients of the two surfaces are the same, $\alpha = 2\alpha^*$. Making these changes, Eq. (4) takes the form

$$Nu_m = 0.143\, Ra_{m_{av}}^{0.3}\, Pr_m^{0.078}. \tag{5}$$

In Fig. 3, curves corresponding to Eq. (5) are compared with curves calculated for the heat transfer under free convection near a vertical wall in a gravitational field [4]. It is evident that at large Pr the heat-transfer coefficients at the rotating surface and at a fixed vertical wall are close, but at small Pr and the same Ra the heat-transfer intensity is significantly less at a rotating surface than at a fixed wall.

The experimental data used in the generalization were obtained for j_c/g = 10–50 and so Eq. (4) may be used without significant error for any spatial orientation of the axis of rotation.

NOTATION

F, mass force per unit volume; N/m^3; F_c, centrifugal force; F_C, Coriolis force; g, acceleration due to gravity, m/sec^2; j_c, centrifugal acceleration, m/sec^2; $Nu^* = \alpha^* R/\lambda$, Nusselt number; $Nu = \alpha R/2\lambda$; Pr, Prandtl number; R, radius of rotating cavity, m; r, radius of rotation, m; $Ra = (\omega^2 R^4/\nu^2)\beta\Delta t Pr$, Rayleigh number; $Ra_{av} = (\omega^2 R^4/2\nu^2)\beta\Delta t Pr$; t_1 and t_2, surface temperatures; $\Delta t = t_1 - t_2$; w, velocity of radial liquid displacement, m/sec; x, coordinate perpendicular to heat-transfer surface; α_1, α_2, mean heat-transfer coefficients at heat-transfer surfaces, W/m^2·deg; β, volume-expansion coefficient, deg^{-1}; λ, thermal conductivity, W/m·deg; ν, kinematic viscosity, m^2/sec; ω, angular velocity, rad/sec.

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MODELING TURBULENT FLOW AT A PERMEABLE PLATE WITH STRONG INJECTION

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It is shown that available experimental data on the thickness of the turbulent-boundary layer and the filling of the velocity profile under strong injection are satisfactorily generalized using the parameter $\rho_W v^2_W/\rho_0 u_0^2$.

The boundary layer at a permeable surface has been investigated in a large number of works but as yet no satisfactory answers have been given to a series of important questions arising from the study of such flows. In particular, there is no agreement as to the existence and form of a universal parameter determining the boundary-layer thickness at a permeable plate under strong injection.

Strong injection is taken to be such that the velocity profile degenerates to a straight line and the concentration of injected material at the permeable wall approaches 100%.

Most researchers believe that the thickness of the turbulent-boundary layer at a permeable plate is uniquely determined by the parameter $(\rho_W v_W/\rho_0 u_0)\cdot \mathrm{Re}^{0.2}$, regardless of the strength of injection. This view is based on experimental results obtained with intermixing gas flows either of the same density or else of only slightly different densities [1-5].

Boundary layers with intermixing gas flows of significantly different densities were first systematically investigated in [2, 3]. It was argued that the generally used injection parameter should be modified by the introduction of a factor with a variable index so as to take into account the density ratio. However, only results obtained for the thermal state of a permeable plate were generalized using this parameter.

Systematic data were given in [6] on the thickness of the turbulent boundary layer at a permeable plate when the intermixing flows are of significantly different densities. It was established that the boundary-layer thickness and the filling of the velocity profile depend on the parameter $(\rho_W v_W/\rho_0 u_0)(\rho_W/\rho_0)^k\cdot \mathrm{Re}^{0.2}$, where $k = -0.25^{\pm 0.05}$ for $1 \le \rho_W/\rho_0 \le 3$ and $k = -0.5^{\pm 0.1}$ for $0.07 \le \rho_W/\rho_0 \le 1$, regardless of the injection strength. It was shown that the same parameter also determines the separation of the boundary layer from the wall.

In [7, 8], an interferometer was used to determine the conditions for the separation of the turbulent-boundary layer at a porous plate with the injection of heterogeneous gas. As a result it was possible to confirm that the separation parameter $(\rho_W v_W/\rho_0 u_0)\cdot \mathrm{Re}^{0.2}$ depends strongly on the density ratio of the intermixing flows.

Thus it has been shown that under strong injection the parameter $(\rho_W v_W/\rho_0 u_0)\cdot \mathrm{Re}^{0.2}$ does not uniquely determine the flow in a turbulent-boundary layer at a permeable plate. Note, however, that empirical parameters with a variable index in the factor $(\rho_W/\rho_0)^k$, such as are introduced in [3, 6], do not have a clear physical meaning, since they derive neither from boundary-layer theory nor from jet theory. Therefore, neither of them can be regarded as a general parameter.

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